Fixing the Locally Optimized RANSAC

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http://cmp.felk.cvut.cz/software/LO-RANSAC/

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Overview

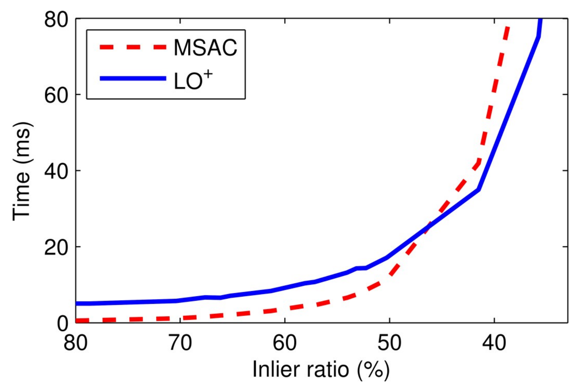
The problem of local optimization for RANSAC is revisited. The following improvements of the LO-RANSAC procedure are proposed:

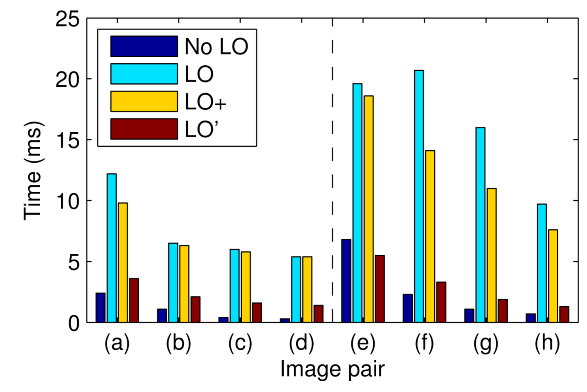
- its (complex) structure validated and tuned,
- the use of a truncated quadratic cost function,
- the use of an inlier limit for the least squares computation,
- several implementation issues were fixed.

Why LO?

- Increases the precision of the returned model.
- Increases the number of inliers and thus less samples are needed.

(1) - (3)





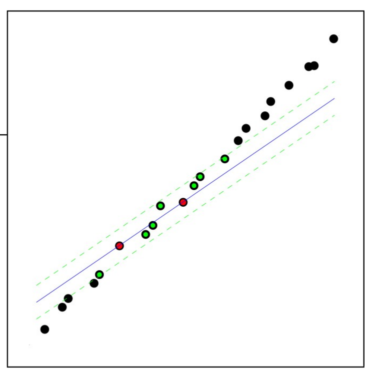
(3) - (5)

The speed-up over LO [1] is typically 10-30% for LO⁺ and up to 6-fold for LO', with negligible effect on the precision.

LO-RANSAC

Algorithm 1 LO-RANSAC.

for $k = 1 \rightarrow \mathrm{K}(|\mathcal{I}^*|, \eta)$ **do**
  $\mathcal{S}_k \leftarrow$ randomly drawn minimal sample
  $M_k \leftarrow$ model estimated from sample $\mathcal{S}_k$
  $\mathcal{I}_k \leftarrow$ *find_inliers*(M_k, θ)
  if $|\mathcal{I}_k| > |\mathcal{I}_s^*|$ **then**
    $M_s^* \leftarrow M_k; \mathcal{I}_s^* \leftarrow \mathcal{I}_k$
    $M_{LO}, \mathcal{I}_{LO} \leftarrow$ run Local Optimization (Alg. 2)
    if $|\mathcal{I}_{LO}| > |\mathcal{I}^*|$ **then**
      $M^* \leftarrow M_{LO}; \mathcal{I}^* \leftarrow \mathcal{I}_{LO}$
      update K
    end if
  end if
end for
return M^*



Algorithm 2 Local Optimization step.

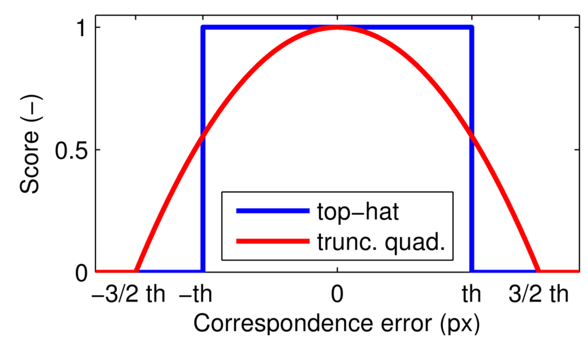
Input: M_s^*, m_θ, *reps*
$M_{m_\theta} \leftarrow$ model estimated by LSq on *find_inliers*$(M_s^*, m_\theta \cdot \theta)$
$\mathcal{I}_{base} \leftarrow$ *find_inliers*(M_{m_θ}, θ)
for $r = 1 \rightarrow$ *reps* **do**
  $\mathcal{S}_{is} \leftarrow$ sample of size s_{is} randomly drawn from $\mathcal{I}_{base}$
  $M_{is} \leftarrow$ model estimated from $\mathcal{S}_{is}$ by LSq
  $M_r \leftarrow$ Iterative Least Squares $(M_{is}, m_\theta, iters)$ (Alg. 3)
end for
return the best of M_s^*, all M_{is}, all M_r, with its inliers

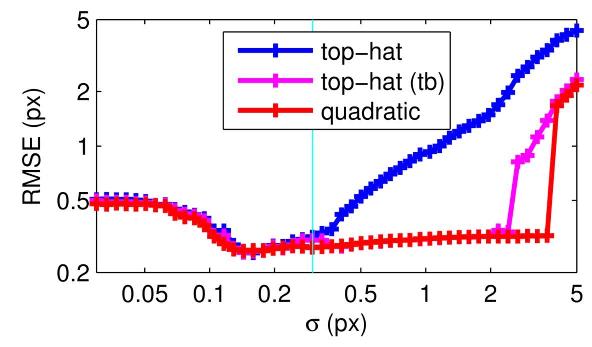
Algorithm 3 Iterative Least Squares.

Input: M_{is}, m_θ, *iters*
$M' \leftarrow$ model estimated by LSq on *find_inliers*(M_{is}, θ)
$\theta' \leftarrow m_\theta \cdot \theta$
for $i = 1 \rightarrow$ *iters* **do**
  $\mathcal{I}' \leftarrow$ *find_inliers*(M', θ')
  $w' \leftarrow$ computed weights of $\mathcal{I}'$ (depend on model)
  $M' \leftarrow$ model estimated by LSq on $\mathcal{I}'$ weighted by w'
  $\theta' \leftarrow \theta' - \Delta_\theta$
end for
return the best M'

Increasing the stability

Top-hat – RANSAC cost function, inlier count (inlier 1, outlier 0).
Truncated quadratic – MSAC cost function.
The top-hat cost function often scores different models with the same score [4] – the quadratic function as a tiebreaker ("tb" note in the graph).





The graphs show the used cost functions and the dependence of an estimation error on the cost function and the error scale (on the "wash" image pair). For its greater stability and robustness to the error scale selection, we use quadratic cost function in further experiments.

Running time of LO-RANSAC

$$t_{tot} = C_R \cdot \mathrm{K} + C_{LO} \cdot \lceil \log(\mathrm{K}) \rceil$$

C_R: time of standard RANSAC hypothesis generation and verification round
K: number of samples drawn
C_{LO}: average time of LO procedure

(1), (3)

For small K (dependent on the inlier ratio), the significant difference of C_{LO} and C_R means that $C_{LO} \cdot \lceil \log(\mathrm{K}) \rceil \gg C_R \cdot \mathrm{K}$, i.e. the vast majority of the time is spent in the local optimization. Therefore, the "negligible extra time for LO" statement by [1] is true only for problems with low inlier ratios.

Speeding up Local Optimization

Time consumption of LO – solving sets of linear equations repeatedly.
Proposed reduction: – to lower the number of equations,
– to lower the number of repetitions.

LO⁺ – a limit on the number of correspondences that participate in the estimation of model parameters (a use of random subset).
LO' – no subsampling, only iterative least squares (with the inlier limit).

Experimental evaluation

Measurements
- Precision
  - Number of inliers
  - Geometric error
- Speed
  - Running time
  - Number of samples
  - Number of LOs

(6), (7)

Non-linear iterative optimization (bundle adjustment, BA) was initialized by the output of MSAC refined by one linear least squares (6), and of fully locally optimized MSAC (7). The latter provides better initialization.

The full dataset contains 16 image pairs for epipolar geometry and 16 pairs for homography estimation. These images were previously used for an evaluation in a number of publications. The dataset is available at: http://cmp.felk.cvut.cz/data/geometry2view/.

Results

Image		(1) MSAC 10 000 runs	(2) MSAC + LSq 10 000 runs	(3) LO 10 000 runs	(4) LO⁺ 10 000 runs	(5) LO' 10 000 runs	(6) MSAC + LSq + BA 100 runs	(7) LO + BA 100 runs
EPIPOLAR GEOMETRY								
(a) Kyoto	Inliers	**295.2** ±16.5 (245-336)	**311.4** ±15.3 (249-339)	**333.5** ±6.7 (274-339)	**330.7** ±5.7 (278-339)	**325.1** ±9.2 (266-340)	**313.7** ±16.7 (267-339)	**332.1** ±8.0 (280-339)
	Inliers (%)	**66.3** ±3.7 (55-76)	**70.0** ±3.4 (56-76)	**74.9** ±1.5 (62-76)	**74.3** ±1.3 (62-76)	**73.0** ±2.1 (60-76)	**70.5** ±3.7 (60-76)	**74.6** ±1.8 (63-76)
	Error (px)	**2.25** ±1.28 (0.3-11.3)	**1.64** ±1.14 (0.3-8.1)	**0.81** ±0.32 (0.4-5.7)	**0.78** ±0.23 (0.3-5.0)	**1.07** ±0.54 (0.3-6.9)	**1.47** ±0.97 (0.4-5.1)	**0.78** ±0.33 (0.5-2.2)
	Time (ms)	**2.4** (NA)	**2.7** (NA)	**12.2** (NA)	**9.8** (NA)	**3.6** (NA)	**18499.7** (NA)	**12006.1** (NA)
	Samples	**65.4** ±26.0 (21-203)	**65.4** ±26.0 (21-203)	**49.2** ±12.1 (21-185)	**49.1** ±12.1 (21-185)	**49.6** ±12.6 (21-185)	**66.8** ±27.5 (25-161)	**51.3** ±14.9 (25-125)
	LO count	**0.0** ±0.0 (0-0)	**0.0** ±0.0 (0-0)	**1.0** ±0.1 (1-2)	**1.0** ±0.0 (1-2)	**1.0** ±0.1 (1-3)	**0.0** ±0.0 (0-0)	**1.0** ±0.0 (1-1)
(b) corr	Inliers	**62.7** ±4.4 (50-76)	**66.0** ±4.2 (46-76)	**73.1** ±1.6 (58-77)	**73.3** ±1.8 (57-78)	**69.8** ±2.8 (53-78)	**67.4** ±4.2 (55-75)	**73.2** ±1.5 (64-76)
	Inliers (%)	**67.4** ±4.7 (54-82)	**71.0** ±4.5 (49-82)	**78.6** ±1.7 (62-83)	**78.8** ±1.9 (61-84)	**75.1** ±3.0 (57-84)	**72.5** ±4.5 (59-81)	**78.8** ±1.7 (69-82)
	Error (px)	**0.48** ±0.33 (0.1-3.0)	**0.37** ±0.33 (0.1-3.4)	**0.18** ±0.11 (0.1-2.7)	**0.18** ±0.10 (0.1-2.0)	**0.31** ±0.12 (0.1-1.9)	**0.34** ±0.25 (0.1-1.7)	**0.16** ±0.04 (0.1-0.4)
	Time (ms)	**1.1** (NA)	**1.3** (NA)	**6.5** (NA)	**6.3** (NA)	**2.1** (NA)	**2459.5** (NA)	**2046.8** (NA)
	Samples	**61.0** ±25.1 (11-211)	**61.0** ±25.1 (11-211)	**49.5** ±15.9 (11-183)	**49.5** ±15.9 (11-183)	**49.7** ±16.1 (11-183)	**63.7** ±27.0 (13-151)	**52.3** ±20.7 (13-136)
	LO count	**0.0** ±0.0 (0-0)	**0.0** ±0.0 (0-0)	**1.0** ±0.0 (1-2)	**1.0** ±0.1 (1-3)	**1.0** ±0.1 (1-3)	**0.0** ±0.0 (0-0)	**1.0** ±0.0 (1-1)
(c) head	Inliers	**66.9** ±4.1 (52-77)	**71.9** ±2.7 (53-76)	**73.9** ±0.6 (69-76)	**74.0** ±0.6 (69-77)	**73.7** ±0.9 (68-76)	**72.9** ±2.0 (66-76)	**74.0** ±0.2 (73-75)
	Inliers (%)	**77.8** ±4.7 (60-90)	**83.6** ±3.1 (62-88)	**86.0** ±0.7 (80-88)	**86.0** ±0.7 (80-90)	**85.7** ±1.0 (79-88)	**84.7** ±2.3 (77-88)	**86.0** ±0.3 (85-87)
	Error (px)	**0.78** ±0.52 (0.2-5.1)	**0.40** ±0.19 (0.2-2.4)	**0.31** ±0.03 (0.2-0.5)	**0.31** ±0.03 (0.2-0.5)	**0.30** ±0.03 (0.2-0.7)	**0.38** ±0.15 (0.3-1.2)	**0.35** ±0.02 (0.3-0.4)
	Time (ms)	**0.4** (NA)	**0.6** (NA)	**6.0** (NA)	**5.8** (NA)	**1.6** (NA)	**812.4** (NA)	**685.8** (NA)
	Samples	**21.8** ±10.1 (5-103)	**21.8** ±10.1 (5-103)	**21.7** ±9.8 (5-103)	**21.7** ±9.8 (5-103)	**21.7** ±9.8 (5-103)	**21.6** ±9.9 (6-50)	**21.6** ±9.9 (6-50)
	LO count	**0.0** ±0.0 (0-0)	**0.0** ±0.0 (0-0)	**1.0** ±0.0 (1-1)	**1.0** ±0.0 (1-1)	**1.0** ±0.0 (1-1)	**0.0** ±0.0 (0-0)	**1.0** ±0.0 (1-1)
(d) wash	Inliers	**45.7** ±3.5 (34-52)	**50.1** ±1.7 (6-52)	**51.3** ±0.4 (51-52)	**51.4** ±0.5 (51-52)	**51.7** ±0.5 (51-52)	**50.6** ±1.0 (47-52)	**51.0** ±0.2 (51-52)
	Inliers (%)	**83.1** ±6.4 (62-95)	**91.1** ±3.1 (11-95)	**93.2** ±0.8 (93-95)	**93.5** ±0.9 (93-95)	**94.0** ±0.8 (93-95)	**92.0** ±1.9 (85-95)	**92.8** ±0.4 (93-95)
	Error (px)	**1.04** ±0.61 (0.2-5.2)	**0.39** ±0.17 (0.2-2.9)	**0.27** ±0.04 (0.2-0.6)	**0.27** ±0.03 (0.2-0.5)	**0.28** ±0.02 (0.2-0.6)	**0.32** ±0.13 (0.2-0.9)	**0.26** ±0.03 (0.2-0.4)
	Time (ms)	**0.3** (NA)	**0.4** (NA)	**5.4** (NA)	**5.4** (NA)	**1.4** (NA)	**132.2** (NA)	**107.4** (NA)
	Samples	**16.7** ±9.8 (3-92)	**16.7** ±9.8 (3-92)	**16.7** ±9.7 (3-72)	**16.7** ±9.7 (3-72)	**16.7** ±9.7 (3-72)	**15.8** ±8.9 (3-43)	**15.8** ±8.9 (3-43)
	LO count	**0.0** ±0.0 (0-0)	**0.0** ±0.0 (0-0)	**1.0** ±0.0 (1-1)	**1.0** ±0.0 (1-1)	**1.0** ±0.0 (1-1)	**0.0** ±0.0 (0-0)	**1.0** ±0.0 (1-1)
HOMOGRAPHY								
(e) Eiffel	Inliers	**60.9** ±4.1 (43-69)	**64.4** ±3.2 (47-70)	**66.8** ±1.1 (62-69)	**66.7** ±1.1 (61-69)	**66.0** ±1.7 (50-70)	**65.3** ±2.6 (55-69)	**66.5** ±0.8 (64-67)
	Inliers(%)	**30.4** ±2.1 (22-34)	**32.2** ±1.6 (24-35)	**33.4** ±0.5 (31-34)	**33.3** ±0.6 (30-34)	**33.0** ±0.9 (25-35)	**32.6** ±1.3 (28-34)	**33.2** ±0.4 (32-34)
	Error (px)	**1.23** ±0.57 (0.3-7.6)	**0.92** ±0.44 (0.3-3.9)	**0.88** ±0.16 (0.6-1.4)	**0.88** ±0.15 (0.5-1.5)	**0.82** ±0.28 (0.3-2.5)	**0.91** ±0.35 (0.6-2.1)	**0.78** ±0.17 (0.6-1.3)
	Time (ms)	**6.8** (NA)	**6.8** (NA)	**19.6** (NA)	**18.6** (NA)	**5.5** (NA)	**28.5** (NA)	**45.1** (NA)
	Samples	**438.9**±155.3(223-1676)	**438.9**±155.3(223-1676)	**254.5**±18.6(223-800)	**254.4**±17.2(210-507)	**273.2**±40.7(210-815)	**444.8** ±168.3(252-1051)	**255.7** ±22.6(237-409)
	LO count	**0.0** ±0.0 (0-0)	**0.0** ±0.0 (0-0)	**2.5** ±1.2 (1-8)	**2.5** ±1.2 (1-9)	**2.6** ±1.2 (1-9)	**0.0** ±0.0 (0-0)	**2.5** ±1.2 (1-8)
(f) Brussels	Inliers	**328.7** ±32.4 (225-394)	**371.4** ±18.2 (264-398)	**390.6** ±1.3 (387-396)	**390.5** ±2.1 (383-397)	**387.9** ±4.4 (345-398)	**379.0** ±8.6 (350-392)	**387.1** ±0.6 (386-388)
	Inliers(%)	**65.3** ±6.5 (45-78)	**73.8** ±3.6 (52-79)	**77.6** ±0.3 (77-79)	**77.6** ±0.4 (76-79)	**77.1** ±0.9 (69-79)	**75.3** ±1.7 (70-78)	**77.0** ±0.1 (77-77)
	Error (px)	**3.65** ±0.92 (2.0-10.6)	**2.59** ±0.50 (0.9-4.8)	**2.88** ±0.05 (2.7-3.0)	**2.86** ±0.08 (2.6-3.1)	**2.25** ±0.20 (1.5-3.2)	**3.15** ±0.21 (2.3-3.6)	**2.97** ±0.01 (2.9-3.0)
	Time (ms)	**2.3** (NA)	**2.6** (NA)	**20.7** (NA)	**14.1** (NA)	**3.3** (NA)	**116.6** (NA)	**112.4** (NA)
	Samples	**21.0** ±9.4 (7-71)	**21.0** ±9.4 (7-71)	**20.9** ±9.2 (7-52)	**20.9** ±9.2 (7-52)	**20.9** ±9.2 (7-52)	**21.8** ±9.6 (8-54)	**21.8** ±9.5 (8-51)
	LO count	**0.0** ±0.0 (0-0)	**0.0** ±0.0 (0-0)	**1.0** ±0.0 (1-1)	**1.0** ±0.0 (1-1)	**1.0** ±0.0 (1-1)	**0.0** ±0.0 (0-0)	**1.0** ±0.0 (1-1)
(g) Boston	Inliers	**277.3** ±21.5 (187-305)	**303.0** ±5.4 (260-305)	**305.0** ±0.0 (305-305)	**305.0** ±0.0 (305-305)	**305.0** ±0.1 (303-305)	**305.0** ±0.2 (304-305)	**305.0** ±0.0 (305-305)
	Inliers(%)	**72.6** ±5.6 (49-80)	**79.3** ±1.4 (68-80)	**79.8** ±0.0 (80-80)	**79.8** ±0.0 (80-80)	**79.8** ±0.0 (79-80)	**79.8** ±0.0 (80-80)	**79.8** ±0.0 (80-80)
	Error (px)	**1.78** ±1.01 (0.4-15.1)	**0.72** ±0.20 (0.4-2.6)	**0.66** ±0.00 (0.7-0.7)	**0.66** ±0.00 (0.6-0.7)	**0.60** ±0.08 (0.3-0.9)	**0.67** ±0.03 (0.6-0.8)	**0.66** ±0.00 (0.7-0.7)
	Time (ms)	**1.1** (NA)	**1.3** (NA)	**16.0** (NA)	**11.0** (NA)	**1.9** (NA)	**82.6** (NA)	**26.0** (NA)
	Samples	**12.8** ±5.8 (6-53)	**12.8** ±5.8 (6-53)	**12.8** ±5.8 (6-50)	**12.8** ±5.8 (6-50)	**12.8** ±5.8 (6-50)	**12.3** ±5.7 (6-38)	**12.3** ±5.7 (6-38)
	LO count	**0.0** ±0.0 (0-0)	**0.0** ±0.0 (0-0)	**1.0** ±0.0 (1-1)	**1.0** ±0.0 (1-1)	**1.0** ±0.0 (1-1)	**0.0** ±0.0 (0-0)	**1.0** ±0.0 (1-1)
(h) WhiteBoard	Inliers	**161.1** ±13.2 (104-174)	**171.6** ±7.7 (135-174)	**174.0** ±0.0 (174-174)	**174.0** ±0.0 (173-174)	**173.7** ±1.8 (137-174)	**172.7** ±6.5 (139-174)	**174.0** ±0.0 (174-174)
	Inliers(%)	**75.3** ±6.2 (49-81)	**80.2** ±3.6 (63-81)	**81.3** ±0.0 (81-81)	**81.3** ±0.0 (81-81)	**81.2** ±0.9 (64-81)	**80.7** ±3.1 (65-81)	**81.3** ±0.0 (81-81)
	Error (px)	**1.48** ±0.49 (0.5-6.0)	**1.09** ±0.19 (0.7-2.7)	**1.08** ±0.00 (1.1-1.1)	**1.06** ±0.01 (1.0-1.2)	**1.02** ±0.06 (0.8-1.9)	**1.08** ±0.12 (1.0-1.7)	**1.05** ±0.00 (1.0-1.0)
	Time (ms)	**0.7** (NA)	**0.8** (NA)	**9.7** (NA)	**7.6** (NA)	**1.3** (NA)	**61.2** (NA)	**53.1** (NA)
	Samples	**11.7** ±5.8 (6-56)	**11.7** ±5.8 (6-56)	**11.7** ±5.8 (6-51)	**11.7** ±5.8 (6-51)	**11.7** ±5.8 (6-51)	**10.2** ±4.3 (6-29)	**10.2** ±4.3 (6-29)
	LO count	**0.0** ±0.0 (0-0)	**0.0** ±0.0 (0-0)	**1.0** ±0.0 (1-1)	**1.0** ±0.0 (1-1)	**1.0** ±0.0 (1-1)	**0.0** ±0.0 (0-0)	**1.0** ±0.0 (1-1)

$\mathbf{X} \pm \sigma_X \, (X_m - X_M)$

mean value ($\mathbf{X}$); standard deviation (σ_X); minimal value (X_m); maximal value (X_M)

Probability of TCs being inliers
x: index of points
y: the probability

Histogram of the probability of TCs being inliers
x: the probability
y: number of points in a bin

(X) see Xth column of the table

Conclusions

LO⁺-RANSAC properties:

- high **stability** (almost non-random algorithm in nature),
- high **precision** in a broad range of conditions,
- lower **sensitivity** to the choice of inlier-outlier threshold and
- it offers a significantly **better starting point** for bundle adjustment (BA) than the Gold Standard method advocated in the Hartley-Zisserman book [3].

The implementation is made publicly available.

References

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